Reliability assessment of the physical modeling of liquefaction-induced effects on shallow foundations considering nonuniformity in the centrifuge model

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ABSTRACT

Physical modeling has been widely used to simulate geotechnical earthquake engineering-related problems and to validate finite element numerical models. In both cases, the model ground is usually considered to have uniform soil properties. However, the model ground is prone to spatial nonuniformity and may affect engineering judgment based on physical modeling. This paper presents a reliability assessment of the physical modeling of liquefaction-induced effects on shallow foundations considering the spatial variability in the centrifuge model. Two-dimensional (2D) finite element simulations with the PM4Sand (version 3.1) elastoplastic soil constitutive model are performed for a sufficient number of stochastic realizations. The nonuniformity in the centrifuge model is implemented with stochastic realizations of the overburden and energy-corrected, equivalent clean sand, SPT (N1)_ec values using a spatially correlated Gaussian random field. The reliability of the centrifuge model test is assessed based on the stochastic average settlement and tilt of the foundation-structure system. The implications of the nonuniformity in the centrifuge model on the liquefaction extent of the ground and spectral displacement of the foundation are also investigated.

1. Introduction

Liquefaction has caused severe damage to environments built on shallow foundations, such as settlement, tilting, and sinking, all over the world during many past earthquakes. Numerous instances of bearing capacity failure of shallow foundations due to liquefaction were observed in the 1964 Niigata and 1990 Luzon (Philippines) earthquakes. Most of the damaged buildings were two to four stories built on shallow foundations with relatively thick and uniform deposits of clean sand [25,29]. Surprisingly, many of the damaged structures were influenced by the liquefaction of thin deposits of silt and silty sand [7,4] in the 1999 Kocaeli (Turkey) earthquake. Many researchers [3,41,44] have also elaborated on the role of liquefaction in the severe damage of buildings, specifically in the reclaimed land off the Pacific coast during the 2011Tohoku Earthquake.

Dynamic centrifuge modeling has been widely used to understand the liquefaction-induced effects on shallow foundations resting on presumably uniform deposits of saturated, loose to dense, and clean sand [46,25,15,29,22]. Several researchers have used dynamic centrifuge model test results to validate their soil constitutive models and finite element numerical models [32,8,17,35,28,26].

Usually, the predicted behavior of shallow foundations resting on the liquefiable ground using the centrifuge model test and the validation of numerical models are based on the assumption that the model ground has uniform soil properties. However, the centrifuge model ground is prone to spatial nonuniformity even though the model ground is intended to be uniformly reconstituted under gravity conditions [36,48]. The nonuniformity in the centrifuge model is evaluated in model scale. However, units in the prototype scale are used in the presented paper to ensure consistency. Several researchers have attempted to obtain the soil's spatial nonuniformity in the centrifuge model ground. For example, Bolton et al. [5] used an in-flight cone penetration test at 70g to obtain insight into the model's spatial nonuniformity. They reported that the nonuniformity in the centrifuge model ground is intended to be uniformly reconstituted under gravity conditions [36,48]. The nonuniformity in the centrifuge model is evaluated in model scale. However, units in the prototype scale are used in the presented paper to ensure consistency. Several researchers have attempted to obtain the soil's spatial nonuniformity in the centrifuge model ground. For example, Bolton et al. [5] used an in-flight cone penetration test at 70g to obtain insight into the model's spatial nonuniformity. They reported that the nonuniformity in the centrifuge model ground is intended to be uniformly reconstituted under gravity conditions [36,48].
ground. They observed the COV of porosity for loose sand and medium dense sand in the range of 1–4% and 1–6%, respectively. The random field approach (details can be found in [33,48]) is adopted to estimate the COV and correlation lengths used in this study considering the random sampling error, effects of container size, and invariability due to method of preparation of the model ground. The estimated COV and the correlation length in the vertical direction are found in the range of 1–6% and 0.5–1.0 m, respectively for the centrifuge experiments at 40g. The estimated correlation length in the horizontal direction is found in the range of 2–6 m. This also reflects the observation of Phoon and Kulhawy [31] and Phoon and Ching [30], which suggests that the correlation length in the horizontal direction is often within an order of magnitude (~10 times) larger than the correlation length in the vertical direction. Different combinations of COV and correlation lengths are considered to trace the average effects of nonuniformity in the centrifuge model, as recommended by Zhang et al. [48].

With the increasing use of centrifuge modeling for the performance prediction of shallow foundations and calibration of numerical models, it is essential to understand the reliability of centrifuge model tests. Reliability analyses provide a means of evaluating the combined effects of uncertainties in the parameters involved in the calculations, and they offer a useful supplement to traditional engineering judgment [16]. For a thorough understanding of risk and reliability analyses in geotechnical engineering, readers are suggested to read Christian et al. [12] and Phoon and Ching [30]. In this paper, an attempt is made to evaluate the reliability of the physical modeling of liquefaction-induced effects on shallow foundations considering the spatial nonuniformity in the centrifuge model. The pouring rate and falling height of Toyoura sand and the pouring direction of the sand hopper are the primary sources of nonuniformity in the centrifuge model. In addition, the size of the model container is limited and a nonuniform model ground along the container boundaries is inevitable.

Centrifuge model test results are used to validate the numerical modeling scheme, which is carried out using the OpenSees (version 3.0.3) framework [27] with the PM4Sand (version 3.1) elastoplastic soil constitutive model [6,9]. Laboratory test results from Chiaro et al. [10,11] are used to calibrate the parameters of the PM4Sand model. The nonuniformity in the centrifuge model is implemented with stochastic realizations of the overburden and energy-corrected, equivalent clean sand, SPT (N1)k0 values using a spatially correlated Gaussian random field [18,42,30]. Two-dimensional finite element simulations are performed for a sufficient number of realizations to account for different combinations of spatial nonuniformity in the centrifuge model, as explained in the subsequent Sections.

2. Physical model

A dynamic centrifuge experiment is carried out to investigate the liquefaction-induced effects on shallow foundations resting on a level deposit of liquefiable Toyoura sand. The centrifuge model contains two shallow foundations and associated superstructures, namely, buffer tank (BT) and flare stack (FS), imposing average bearing pressures of 51.2 kPa and 71.2 kPa, respectively, at 0.8 m below the surface of the model ground in the prototype scale, as shown in Fig. 1. Dynamic centrifuge model test is carried out utilizing the Tokyo Tech Mark III centrifuge facility [40] with a radius of 2.45 m, at a centrifugal acceleration of 40g (N = 40). The centrifuge model test simulates a prototype saturated soil deposit of a depth of 10 m, with a water table located 1.8 m below the top surface. The model ground is prepared using Toyoura sand with a target relative density of 50% by the air pluviation method using a sand hopper with a nozzle outlet. Multiple transducers (e.g., pore pressure transducers, accelerometers, laser displacement transducers, and potentiometers) are carefully placed at desirable locations during the model ground preparation. More details about the physical modeling of the liquefaction-induced effects on the shallow foundation and the comprehensive interpretation of the centrifuge model test results can be obtained from Kumar et al. [23].

3. Numerical model

Initially, the numerical tool is validated against the centrifuge model test before performing a series of stochastic analyses to investigate the reliability of the physical modeling of liquefaction-induced effects on shallow foundations. The numerical model is then used to conduct a series of simulations to assess the effects of spatial nonuniformity on the behavior of the model ground. More details about the numerical modeling approach and the results of the simulations can be found in Kumar et al. [23].
induced effects on shallow foundations considering the spatial non-uniformity in the centrifuge model. Half of the centrifuge model configuration, i.e., the buffer tank and the associated foundation, is considered for the numerical simulations, as shown in Fig. 2. Numerical simulations are carried out with a 2D plane strain solid-fluid fully coupled analysis approach. Rayleigh damping of 1% at a frequency of 1 Hz, corresponding to the first-mode of a typical nonlinear ground response, is used in the analyses.

The model ground is modeled using quadrilateral u-p (quadUP) elements [45]. The footing is modeled using quadrilateral (quad) elements. The bottom nodes of the model ground are kept fixed in both degrees of freedom. The displacement time series of the Tokachi-Oki ground motion (NS component of the recorded shaking at the Hachinohe Port in 1968) is imposed on the bottom nodes of the model ground during dynamic analyses using the multiple support excitation technique. The footing elements are connected to the model ground using the equal degrees of freedom (equalDOF) technique in OpenSees. The side nodes of the model ground are connected using equalDOF to ensure laminar behavior during the dynamic analyses. All the nodes above the water table are assigned a pore water pressure of zero. The efficacy of mesh size is ensured before performing the numerical analyses. The maximum size of the element at any depth is calculated to ensure the proper wave propagation with respect to the minimum wavelength corresponding to the small-strain shear wave velocity profile of the ground and the maximum frequency of the input ground motion after filtration (bandpass 0.10–15 Hz) and baseline correction. The reduction in shear wave velocity due to soil-softening during liquefaction is accommodated with a factor of safety equal to five. In addition, the numerical results (average settlement and tilt of footing) for 50% coarser sand are considered to have a factor of safety equal to five. In addition, the numerical results (average settlement and tilt of footing) for 50% coarser and finer mesh (by length) do not show the significant change in the results corresponding to the adopted mesh.

The PM4Sand soil constitutive model is used to capture the dynamic behavior of the model ground during shaking. PM4Sand is a stress-ratio controlled, critical state compatible, bounding surface plasticity model developed for earthquake engineering applications [6]. This constitutive model requires the specification of three primary input parameters, all of which are dimensionless: the apparent relative density (DR), which controls the dilatancy and stress-strain response; the shear modulus coefficient (G0), which controls the small-strain shear modulus; and the contraction rate parameter (hpo), which is used to adjust the contraction rate to achieve the target cyclic resistance ratio. The calibrated values of G0 and hpo for the deterministic analysis (with uniform ground) with DR = 50% are 347.2 and 0.03, respectively. A detailed description of the secondary parameters and their default values can be obtained from Boulanger and Ziotopoulou [6].

The parameters of the PM4Sand Model are calibrated to achieve a single-amplitude shear strain of 3% during cyclic undrained simple shear loading with an initial static shear stress ratio of zero on a horizontal plane at a single element level within 14.5–15.5 cycles. It is to be noted that the model’s parameters are calibrated at a single element level, and the response is accepted at the system level. The primal reason for this is that the soil response change with the density which has to be properly modeled in the stochastic analyses. In addition, the parameters are calibrated to ensure that the model exhibits similar cyclic mobility, a similar accumulation rate of the shear strain, and similar small strain shear modulus at a single element level, as observed in the laboratory tests. Laboratory test results from Chiaro et al. [10,11] are considered for the dynamic behavior of saturated Toyoura sand with a relative density of 50% at a single element level for the generalized calibration of the PM4Sand model’s parameters.

Fig. 3(a) shows a typical response of the calibrated PM4Sand Model for a cyclic stress ratio (CSR) = 0.178, DR = 50%, and σ′vc = 100 kPa during cyclic undrained simple shear loading with an initial static shear stress ratio of zero on a horizontal plane. The PM4Sand model exhibits the ability of shear strain accumulation commonly referred to as cyclic mobility, which is evident from the stress-strain behavior. The stress path is shown in Fig. 3(b). In the first cycle of loading, the vertical effective stress ratio quickly decreases to 80%. After the vertical effective stress ratio decreases to 40%, large shear strains are triggered (as shown in Fig. 3(a)), and the vertical effective stress ratio decreases to nearly zero within a few cycles. The numerically simulated cyclic response at the single element level is obtained after calibrating the parameters of the PM4Sand model to achieve a similar response as observed in the experiment in terms of the cyclic mobility, initial shear modulus, and accumulation rate of the shear strain. Fig. 3(c) shows the CSR curves corresponding to single-amplitude shear strains of 3% with an initial static shear stress ratio of zero. It should be noted that each
loading cycle is divided into four quarters. For instance, the 10, 10.25, 10.50, and 10.75 cycles mean that the single-amplitude shear strain of 3% is achieved in the first, second, third, and fourth quarters of the 10th cycle, respectively, for a corresponding CSR. It is evident from Fig. 3(c) that the PM4Sand model can map the CSR behavior of Toyoura sand as obtained in the experiment with good agreement.

The ability of the numerical model is examined through the simulation of the liquefaction-induced effects on a shallow foundation at the system level. The capabilities of the PM4Sand model for simulating the dynamic behavior of saturated liquefiable ground at the single element level have been validated using Fig. 3. Time histories of the measured pore pressure, acceleration, and displacement at several locations (as shown in Fig. 2) are compared with the respective numerically simulated time histories. Fig. 4 shows the measured and simulated time histories of the excess pore pressure. It is evident that the PM4Sand model can map the overall trend of the excess pore pressure evolution at all locations except that the model exhibits a relatively slower rate of generation of the excess pore pressure in the early phase of shaking. Moreover, the PM4Sand model is also able to capture the maximum magnitude of the excess pore pressure with good agreement.

Fig. 5 shows the measured and simulated acceleration time histories along with the computed spectral acceleration ratio. The PM4Sand model shows the marginal attenuation in the acceleration time history at A3 in comparison with the trend observed in the centrifuge model test in the early phase of shaking (before 20 s). The seismic performance of the foundation-structure system on the liquefiable ground significantly depends on the low-frequency component of input shaking. The attenuation or amplification of input wave primarily governed by the liquefaction extent of the ground which is influenced by the non-uniformity in the centrifuge model. Although the numerical model shows the somewhat larger spectral acceleration ratio for the high-frequency content, the Fourier amplitude of the input shaking in that frequency range is small from the beginning and the difference in the acceleration ratio for the high-frequency range has less impact on the settlement behavior of the structure for liquefaction-related problems. The spiky behavior in the later stage of shaking is caused by the soil dilation and re-stiffening mechanism of the stress-strain curve for the PM4Sand model.

The simulated and measured displacement time histories of the footing are compared in Fig. 6. The simulated rate of the vertical displacement of the footing before 40 s is relatively large in comparison with the measured rate in the centrifuge model test. The settlement progression after shaking is evident for the case of the measured footing settlement, whereas the numerical model does not show such a tendency. The shear-induced settlement and the settlement caused by re-consolidation strains due to simultaneous partial drainage govern the overall evolution of the footing settlement and tilt measured in the centrifuge model test. However, the numerical model seems to overestimate the shear-induced settlement and significantly underestimate the settlement caused by re-consolidation. Several researchers have made similar observations, e.g., Taibet et al. [39], Dashti and Bray [14], and Karimi and Dashti [20,21]. The numerical models typically exhibit limitations in capturing the settlement caused by partial drainage and re-consolidation during and after shaking because of the characteristics of their constitutive formulations, as reported by Shahir et al. [37], Boulanger and Ziotopoulou [6], Karimi and Dashti [21], and Adamidis and Madabhushi [1]. Overall, it can be said that the simulated displacement time histories are comparable to the measured ones. In addition, the numerical model can capture the total settlement and the tilt of the footing at the end of the shaking, which is further used for the stochastic investigation, as discussed in subsequent sections.

4. Stochastic model

The nonuniform relative density within the centrifuge model ground is mapped using the overburden and energy-corrected, equivalent clean sand, SPT (N1)60cs values as suggested by Montgomery and Boulanger [28]. A series of two-dimensional stochastic dynamic analyses are performed considering the centrifuge model ground properties based on anisotropic, spatially correlated Gaussian random fields of (N1)60cs values. A Gaussian correlation function is used, and the random field is generated through LU decomposition of the covariance matrix as per Constantine and Wang [13]. The PM4Sand model has three primary input parameters (DR, G0, hpo), which can be calibrated (along with the secondary input parameters) per the randomly generated (N1)60cs values. For a given (N1)60cs value, the relative density (DR) and parameter G0 are computed as follows:

\[
D_R = \sqrt{\frac{(N1)_{60cs}}{46}}
\]

\[
G_0 = \left(\frac{G_{\text{max}}}{P_{\text{a}}}\right)^{0.5}
\]

where \( P' \) is the mean effective stress and \( P_{\text{a}} \) is the atmospheric pressure. The value of \( G_{\text{max}} \) is computed using the correlation proposed by Andrus and Stokoe [2] for a soil shear wave velocity (Vc) with a slight modification [28] as follows:

\[
G_{\text{max}} = \rho (V_c)^3
\]

\[
V_c = 85[(N1)_{60cs} + 2.5]^{0.25} \left(\frac{P}{P_{\text{a}}}\right)^{0.25}
\]

where \( \rho \) is the mass density of the ground, which is assigned a uniform value of 1.92 ton/m^3 in the present study. The whole model ground is assigned a uniform permeability value of 0.0002 m/s. More index properties of Toyoura sand can be obtained from Kumar et al. [23]. The last primary input parameter (hpo) is calibrated to achieve a single-amplitude shear strain of 3% during cyclic undrained simple shear loading with an initial static shear stress ratio of zero on a horizontal plane at the single element level. The random field of (N1)60cs values with calibrated parameters of the PM4Sand model are implemented.
into the OpenSees numerical model with the help of MATLAB code. Eighteen different cases of nonuniformity in the centrifuge model are considered as tabulated in Table 1, and a total of forty realizations are generated for each of the cases. The number of realizations is determined based on the convergence of the mean and standard deviation of the average footing settlement and tilt. All the cases have a mean value of \((N_1)_{60cs} = 12\) (DR ~ 50%) with different combinations of nonuniformity in the centrifuge model. The tabulated coefficient of variation (COV) and scale of fluctuation (\(\theta_x\) and \(\theta_y\)) are considered according to Bolton et al. [5], White et al. [43], Li et al. [24], and Zhang et al. [48] as described in the introduction Section.

A typical spatial distribution of \((N_1)_{60cs}\) values for COV = 6%, \(\theta_x = 2\) m, and \(\theta_y = 1\) m for mapping the nonuniformity in the centrifuge model is shown in Fig. 7. Fig. 7(a) shows the contours of the \((N_1)_{60cs}\) values within the model ground for a typical realization. The cumulative probability distributions of all forty realizations are shown in Fig. 7(b). The values of \((N_1)_{60cs}\) vary between 10 and 14 for all realizations with different probabilities of occurrence. Fig. 7(c) shows that the generated spatial distribution of the \((N_1)_{60cs}\) values can be fitted with a Gaussian normal distribution with a specified mean (\(\mu\)) and standard deviation (\(\sigma\)).

### Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean ((\mu)), ((N_1)_{60cs})</th>
<th>COV (%)</th>
<th>(\theta_x) (m)</th>
<th>(\theta_y) (m)</th>
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<tr>
<td>A</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
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<tr>
<td>B</td>
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<td>C</td>
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<td>R</td>
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COV: coefficient of variation, \(\theta_x\) and \(\theta_y\): correlation length in X and Y direction (see Fig. 2) respectively.

A typical spatial distribution of \((N_1)_{60cs}\) values for COV = 6%, \(\theta_x = 2\) m, and \(\theta_y = 1\) m for mapping the nonuniformity in the centrifuge model is shown in Fig. 7. Fig. 7(a) shows the contours of the \((N_1)_{60cs}\) values within the model ground for a typical realization. The cumulative probability distributions of all forty realizations are shown in Fig. 7(b). The values of \((N_1)_{60cs}\) vary between 10 and 14 for all realizations with different probabilities of occurrence. Fig. 7(c) shows that the generated spatial distribution of the \((N_1)_{60cs}\) values can be fitted with a Gaussian normal distribution with a specified mean (\(\mu\)) and standard deviation (\(\sigma\)).

Fig. 5. Acceleration response (a) measured and simulated acceleration time histories during Tokachi-Oki ground motion and (b) computed spectral acceleration ratio and Fourier spectrum of applied base shaking.

Fig. 6. Measured and simulated footing settlement during Tokachi-Oki ground motion.
Fig. 8 shows the typical variation of the mean and standard deviation of the average footing settlement \(((\text{LDT}_1 + \text{LDT}_2)/2)\) and tilt \(((\text{LDT}_1 - \text{LDT}_2)/W; W = 4.0 \text{ m}, the width of the footing)\). The mean and standard deviation become stable within forty realizations, and hence, a reliable statistical interpretation of the stochastic data can be obtained from the series of nonlinear dynamic numerical simulations. It should be noted that the greater the number of realizations, the better the reliability of the statistical interpretation. However, the numerical computational expense should be taken into account when selecting the total number of realizations without compromising the stability of the mean and standard deviation of the primary stochastic outcomes (e.g., the average settlement and tilt of the footing presented in the paper).

5. Results and discussion

5.1. Average settlement and tilt of the footing

Two-dimensional stochastic analysis results are presented and compared with the deterministic analysis results in the prototype scale. In the case of deterministic analysis, uniform model ground is assumed with \((N1)_{60cs} = 12 (D_0 \sim 50\%)\). The deterministic analysis is initially validated with a dynamic centrifuge model test, as explained in Section 3. Fig. 9 illustrates the stochastic distribution of the average footing settlement for different combinations of nonuniformity in the centrifuge model. The average footing settlement is calculated by taking the average of the readings of LDTs 1 and 2 (the location of LDTs are shown in Fig. 2) at the end of the shaking. The mean \((\mu)\) and the standard deviation \((\sigma)\) of the average footing settlement are found in the ranges of 4.90–5.12 cm and 0.294–0.508 cm, respectively. It is evident that the mean values of stochastic average footing settlement for different combinations of nonuniformity in the centrifuge model (as tabulated in Table 1) are comparable to the deterministic values of the average footing settlement. However, a relatively wide range of standard deviations cannot be ignored, and the implications of the atypical distributions of the average footing settlement (as shown in Fig. 9) are subsequently discussed with the help of Fig. 11.

Fig. 10 shows the stochastic distribution of the footing tilt for different combinations of nonuniformity in the centrifuge model. The footing tilt is calculated using the readings of LDTs 1 and 2 as \((\text{LDT}_1 - \text{LDT}_2)/4.0 (the width of the footing is 4.0 \text{ m}, as shown in Fig. 2) at the end of the shaking. The mean \((\mu)\) and the standard deviation \((\sigma)\) of the footing tilt are found in the range of 0.0021–0.0029 rad and 0.0011–0.0023 rad, respectively. It is evident that the mean stochastic footing tilt for different combinations of nonuniformity in the centrifuge model is significantly larger than the deterministic value of the footing tilt. This notable difference (with a
maximum value of $0.0029 - 0.0013 = 0.0016$ rad, which is even more than the deterministic value of $0.0013$ rad in the stochastic mean and deterministic value of the footing tilt suggests that the nonuniformity in the centrifuge model has a significant impact on the tilt of the footing. It should be noted that all of the stochastic distributions of the footing tilt with different combinations of centrifuge model nonuniformity are positively skewed from the deterministic value of the footing tilt, as shown in Fig. 10. This emphasizes that the deterministic numerical simulation (with uniform ground properties) substantially underestimates the tilt of the footing. The observations from Figs. 10 and 11 echo the general notion that the deterministic analyses underestimate the settlement and tilt of the footing. However, the probability of their occurrence must be determined, as shown in Fig. 11.

The probability of deviation of the stochastic average footing settlement and tilt from their deterministic values are evaluated and presented in Fig. 11 for different combinations of non-uniformity in the centrifuge model. The deviations of the average footing settlement and tilt are considered on the positive side (more than the deterministic value) and negative side (less than the deterministic value). The maximum deviation of the average footing settlement and footing tilt determined from their deterministic values, along with the associated probability of occurrence, are tabulated in Table 2 for the ease of interpreting Fig. 11. The probability of the average footing settlement being less than the deterministic value is found in the range of 28.06–39.20%. The maximum deviation of the average footing settlement on the negative side is found in the range of 0.47 cm (with a 4.37% probability of occurrence) to 0.90 cm (with a 3.04% probability of occurrence). However, the probability of the average footing settlement being more than the deterministic value is found in the range of 60.80–70.94%. The maximum deviation of the average footing settlement on the positive side is found in the range of 0.90 cm (with a 2.96% probability of occurrence) to 1.51 cm (with a 0.04% probability of occurrence). The probability of the footing tilt being less than the deterministic value is found in the range of 14.84–20.71%. The maximum deviation of the footing tilt in the negative side is found in the range of 0.0016 rad (with a 1.80% probability of occurrence) to 0.0032 rad (with a 2.20% probability of occurrence). However, the probability of the footing tilt being more than the deterministic value is found in the range of 70.30–85.16%. The maximum deviation of the footing tilt in the positive side is found in the range of 0.0016 rad (with a 1.80% probability of occurrence) to 0.0032 rad (with a 2.20% probability of occurrence). However, the probability of the footing tilt being more than the deterministic value is found in the range of 70.30–85.16%. The maximum deviation of the footing tilt in the positive side is found in the range of 0.0043 rad (with a 1.57% probability of occurrence) to 0.0077 rad (with a 0.06% probability of occurrence). These statistics signify that unlike the average footing settlement, the footing tilt is prone to have a significant deviation from the deterministic value with a relatively large probability of occurrence.
5.2. Expected error

The numerical model (Fig. 2) is an idealized abstraction of the centrifuge model (Fig. 1). Hence, the model uncertainty may affect the reliability of stochastic analyses [47]. A non-dimensional (normalized) root-mean-square error is calculated for the average settlement and tilt of the footing to trace the severity of the error induced due to model uncertainty under the assumption of the random sampling of non-uniformity as reported by Popescu and Prevost [33]. The expected error ($\varepsilon_n$) for random realizations can be calculated as follows:

$$\varepsilon_n = \frac{\sigma_n}{\mu_n \sqrt{n}}$$  \hspace{1cm} (5)

where $\sigma_n$ and $\mu_n$ are the standard deviation and mean of the stochastic average settlement and tilt of the footing, respectively, for n realizations.

Fig. 12 shows the expected error magnitude in the estimation of the average settlement and tilt of the footing for different combinations of non-uniformity in the centrifuge model. The expected error is compared with the maximum allowable error ($\varepsilon_{max} = 0.35/\sqrt{n} = 0.055$, for n = 40 realizations, [33]). A scattered trend in expected error magnitude is observed until 15 and 20 realizations for the average settlement and tilt of the footing, respectively. It is evident that the expected error magnitude is significantly large for fewer realizations (n < 10). The footing tilt is prone to have a large expected error magnitude in comparison with the average footing settlement. The observed trends of the expected error magnitude are consistent with those reported by Popescu and Prevost [33] and [34]. The expected error magnitude decreases with increasing number of realizations, having a notable margin from the maximum allowable error for a total of forty realizations. This also confirms that forty realizations are sufficient for the reliable statistical interpretation of stochastic data, as explained in Section 4.

5.3. Displacement response spectra

The displacement time history of the input motion (Tokachi-Oki) is applied at the base of the numerical model. The frequency and magnitude of the input shaking fluctuate (amplify or attenuate depending upon the soil-structure interaction) as the wave propagates toward the surface of the ground. The response of the superstructure significantly depends on the characteristics of the shaking at the foundation. An attempt is made to understand the stochastic response of the foundation-structure system in terms of the spectral displacement for different combinations of nonuniformity in the centrifuge model. For each realization, the displacement time history of the footing is recorded during shaking. Then, the spectral displacement (horizontal) is calculated for a wide range of fundamental periods ($T = 0.0005–4$ s), considering a damping ratio of 5%. A liquefied ground usually filters the high-frequency content of the incident wave while amplifying the magnitude of the low-frequency content. The amplification in the magnitude of the low-frequency content of the incident wave has a significant impact on the spectral displacement of the foundation-structure system.

Fig. 13(a) depicts the mean spectral horizontal displacement of the footing against a wide range of fundamental periods along with the mean ($\mu$) ± standard deviation ($\sigma$) trends. It is found that the spectral
displacement starts to deviate from its mean value for periods of more than 0.7 s. This emphasizes that the consideration of nonuniformity in the centrifuge model is essential for structures with long fundamental periods. A total of forty realizations are carried out for each case of nonuniformity in the centrifuge model, as discussed earlier in Section 4. The spectral displacement corresponding to each realization for long periods, T = 2, 3, and 4 s (for the sake of brevity, only three periods are selected) is used to exhibit the stochastic distributions of the spectral displacement as shown in Fig. 13(b). The spectral displacement significantly deviates from its mean value with a significant standard deviation. This emphasizes that the consideration of nonuniformity in the centrifuge model is vital to evaluate the seismic behavior of the foundation-structure system.

5.4. Liquefaction potential index

An attempt is made to evaluate the severity of the liquefaction-induced impact on the foundation-structure system in correlation with the average footing settlement and tilt for different combinations of nonuniformity in the centrifuge model. A liquefaction potential index ($I_L$) is calculated per Iwasaki et al. [19] and Sonmez and Gokceoglu [38] with a slight modification as follows:

$$I_L = \int_0^Z F(10 - 0.5Z)dz, \quad Z \leq 20 \text{ m}$$

where Z is the depth of the ground (=10 m in this study), and F is defined as the ratio of the area of the liquefied elements and the total...
### Table 2
The probability of deviation of the stochastic average footing settlement and tilt from their deterministic values. A few numbers extracted from Fig. 11 are tabulated in this table.

<table>
<thead>
<tr>
<th>Nonuniformity in the centrifuge model</th>
<th>Average footing settlement</th>
<th>Footing tilt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Negative* side (cm)</td>
<td>Probability (%)</td>
</tr>
<tr>
<td>COV = 2%, θ_y = 0.5 m</td>
<td>-ve 38.21</td>
<td>+ve 61.79</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.60 1.18</td>
<td>0.97 0.87</td>
</tr>
<tr>
<td>COV = 4%, θ_y = 0.5 m</td>
<td>-ve 34.84</td>
<td>+ve 65.16</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.48 5.39</td>
<td>1.19 0.19</td>
</tr>
<tr>
<td>COV = 6%, θ_y = 0.5 m</td>
<td>-ve 32.40</td>
<td>+ve 67.60</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.47 4.37</td>
<td>1.51 0.04</td>
</tr>
<tr>
<td>COV = 2%, θ_y = 1.0 m</td>
<td>-ve 28.06</td>
<td>+ve 70.94</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.48 6.31</td>
<td>0.91 0.92</td>
</tr>
<tr>
<td>COV = 4%, θ_y = 1.0 m</td>
<td>-ve 39.20</td>
<td>+ve 60.80</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.90 3.04</td>
<td>0.90 2.96</td>
</tr>
<tr>
<td>COV = 6%, θ_y = 1.0 m</td>
<td>-ve 30.26</td>
<td>+ve 69.74</td>
</tr>
<tr>
<td>θ_x = 2 m, 4 m, and 6 m</td>
<td>-0.74 0.77</td>
<td>1.27 0.44</td>
</tr>
</tbody>
</table>

* −ve is less than the deterministic value.
** +ve is more than the deterministic value.

---

Fig 12. Expected error magnitude for different combinations of nonuniformity in the centrifuge model (average values for COV = 2%, 4%, and 6%): (a) average footing settlement and (b) footing tilt.

Fig 13. Response of the foundation-structure system for a typical case of nonuniformity in the centrifuge model (COV = 6%, θ_x = 4 m, and θ_y = 0.5 m): (a) spectral displacement with mean (μ) ± standard deviation (σ) and (b) distributions of the spectral displacement at periods of 2, 3, and 4 s.
area of the elements under the footing at a depth of Z. An element (mesh is shown in Fig. 2) is considered to be liquefied if the excess pore pressure ratio is more than or equal to 0.9. The excess pore pressure ratio (ru) is defined as the ratio of the excess pore pressure to the initial vertical effective stress. In the original liquefaction potential index [19], F is the factor of safety against liquefaction defined as FL. Since FL cannot be obtained explicitly from the calculation, it is replaced with the proportion of the liquefied soil in this study.

Fig. 14(a) shows that the mean and the standard deviation of IL (for a typical case of nonuniformity in the centrifuge model with COV = 6%, $\theta_x = 4\,\text{m}$, and $\theta_y = 0.5\,\text{m}$) become stable within forty realizations; hence, a reliable statistical interpretation of the impact of the severity of ground liquefaction on the behavior of the foundation-structure system can be made. Fig. 14(b and c) show the stochastic correlation between IL and the average footing settlement and the tilt. Nearly 90% of the IL values are found in the range of 8–18, corresponding to an average footing settlement in the range of 4.28–5.46 cm with a few (~10%) scattered values in the range of 18–28. However, nearly 87% of the IL values are found in the range of 8–18, corresponding to a footing tilt in the range of 0.0010–0.0052 rad, with a few (~13%) scattered values in the range of 18–28.

Fig. 15 shows the overall range of the average settlement and tilt of the footing with a 95% confidence level considering the different combinations of nonuniformity in the centrifuge model. It can be observed that the stochastic mean values of average footing settlement for different combinations of nonuniformity in the centrifuge model are comparable to the deterministic values. However, the 95% confidence range of the footing tilt significantly deviates from its deterministic value. This observation signifies that the deterministic numerical simulation (with uniform ground properties) substantially underestimates the tilt of the footing, and the footing tilt is prone to be severely affected by the nonuniformity in the centrifuge model.

6. Conclusions

A reliability assessment of the physical modeling of liquefaction-induced effects on shallow foundations considering nonuniformity in the centrifuge model is carried out using two-dimensional (2D) stochastic numerical analyses. The numerical modeling scheme is validated at the element level and at the system level by simulating the centrifuge model test, which is performed to investigate the liquefaction-induced effects on the shallow foundation. The PM4Sand elasto-plastic soil constitutive model is used to simulate the dynamic behavior of the liquefiable model ground. The nonuniformity in the centrifuge model is mapped with the stochastic realizations of the overburden and energy-corrected, equivalent clean sand, SPT (N1)60cs values using a spatially correlated Gaussian random field. The nonuniformity in the centrifuge model is found to influence the engineering judgment made from the centrifuge model test for various types of problems, such as the average footing settlement and tilt, liquefaction severity of the ground, and implications of the ground-foundation-structure interaction. The stochastic average footing settlements with different combinations of centrifuge model nonuniformity are comparable to the deterministic average footing settlement. However, the nonuniformity in the centrifuge model is found to have a significant impact on the tilt of the footing. It is observed that the deterministic numerical simulation (having uniform ground properties) significantly underestimates the tilt of the footing. Stochastic results also indicated that the liquefaction

Fig 14. Liquefaction severity and the response of the foundation-structure system: (a) a typical convergence check for the liquefaction potential index (IL), (b) correlation between the average footing settlement and IL, and (c) correlation between the footing tilt and IL.

Fig 15. Stochastic range (with 95% confidence level) of the average settlement and tilt of the footing.
extent in the model ground varies with the centrifuge model’s non-uniformity and is correlated with the effects on the foundation-structure system. The stochastic displacement spectra exhibited that the nonuniformity of the centrifuge model ground should be taken into account, especially for long-period structures. The reliability assessment of the centrifuge model test results is essential for better engineering judgment associated with a desired level of confidence. The presented probabilistic correlations between nonuniformity of the centrifuge model and the response of foundation-structure system possess significant practical importance and provides useful information to assess the reliability of the physical model tests by numerical procedure. For a generalized framework to incorporate the reduction in the water table, and ground conditions.

CRediT authorship contribution statement

Ritesh Kumar: Conceptualization, Methodology, Investigation, Data curation, Writing - original draft. Kyonobu Kasama: Writing - review & editing. Akihiro Takahashi: Supervision, Conceptualization, Methodology, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References


